Assessment of Skid Resistance in Chilean Road Network

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ABSTRACT

During road network management, is essential to evaluate and know skid resistance levels and their seasonal behavior. With this information, it is possible to develop time dependent models of skid resistance behavior, define skid resistance thresholds and establish skid resistance policies to improve road safety.

Since the year 2005, the Pontificia Universidad Catolica de Chile, The Ministry of Public Works and some private concessions, have been carrying out a skid resistance survey of the road network with SCRIM device. The scope of this is to establish a knowledge basis for the study of the long term and seasonal behavior of Skid Resistance, and to define skid resistance thresholds for Chile.

The surveys have been performed in the Central zone of Chile, in test sections of concrete, asphalt and double treatment surfaces. To analyze time-dependent behavior, a "windows sampling method" was developed. To analyze the seasonal effect over skid resistance, surveys were performed in every season of the year.

This paper presents the results of the statistical analysis of the skid resistance data collected in the field, considering a detailed analysis of the observed behavior. In the first part of the paper, experimental and conceptual issues are presented. In the second part, the long term behavior is analyzed using analytical models. And in the third part, seasonal effect over skid resistance is discussed.

From the analysis it was concluded that a two - stage conceptual model to explain the long term behavior of skid resistance in Chile is not suitable, because traffic is not enough to start the polishing of the aggregates in early ages of the pavement. Also, the statistical analysis of seasonal effects showed that during spring and summer, the seasonal effect mainly affect asphalt surfaces.

Keywords: SKID RESISTANCE – MACRO TEXTURE – TRAFFIC EFFECT – SEASONAL EFFECT – MODELING – ASPHALT – CONCRETE – DOUBLE TREATMENT.

1. INTRODUCTION

The adherence between the tire and the pavement can be characterized by mean of skid resistance forces. Skid resistance force is the result of the combination of adhesion, hysteresis, viscoelastic and molecular break forces. All these forces acts when the pneumatic interacts with the pavement surface in dry or wet environment.

Several studies has shown that lower values of skid resistance in pavement surfaces are related with skidding accidents, specially in pavement wet conditions. Therefore, a desirable goal to achieve in a highways management contexts is to keep higher values of skid resistance in a pavement surface over the time.

To manage skid resistance is necessary to know the factors that affect their value and how it behaves over the time. On this way, countries like United Kingdom, Spain and some Transport State Agencies of the United States developed periodic surveys and built data bases of records of skid resistance and macro texture. In Chile, from the year 2000 to the present, the National Highways Laboratory (NHL) develop surveys only in concessed highways to control the accomplishment of single fixed thresholds of skid resistance.

Between the year 2005 and 2008, the Pontificia Universidad Catolica and the HNL carry on a project for survey and analyze the behaviour of skid resistance in the national paved road network. The objective was to obtain a knowledge base to study how the skid resistance behaves over the time considering long term and seasonal behaviours and to support the update of skid resistance thresholds in Chile. In the surveying, the SCRIM device of the HNL was used. The survey was the first long scale experiment in South America to research the skid resistance behaviour.

In this paper, the authors expose the results of the analysis of the skid resistance data collected and the trends observed. To do this, the follow subjects are discussed:

- Experimental Design: This step considers the planning of measurements data collection, measurements and data processing. The sample size was estimated using the statistic power method proposed by Cohen (1988). Individual data bases of pavement road inventory, traffic, skid resistance measurements and macro texture were integrated on a master data base. On the basis of the experience obtained during the study a measurement and processing method was proposed.

- Exploratory Data Analysis (EDA). This step consider the basic statistical analysis of data to identify trends and general behaviour of skid resistance and macro texture data. In addition, the weak ergodic hypothesis was verified to validate the sample method used in this work.

- SR behaviour Modelling: In this step a conceptual model of skid resistance behaviour was built first structured in sub-models, linked one to each one. Each individual models was calibrated and validated to obtain the final model that describes the skid resistance trend over the time and their seasonal behaviour. To calibrate the models the non-linear robust method and cluster analysis tool was used.

2. THE EXPERIMENT

Experimental design had the objective of define the explanatory variables, an arrangement of hypothetic interaction between them and to estimate the number and location of test sections in which to perform the measurements. This chapter describes the planning of the experiment performed, the data collection procedure and the preliminary statistical analyses of the data collected at a network level.

2.1 PLAN OF MEASUREMENTS

To design the plan of experiment, the principles of statistic experimental design was used. The variables considered in this study were obtained by mean of an exhaustive analysis of the state of arts about modelling skid resistance behaviour study developed by De Solminihac et al (2006).

Considering that study, the following variables were selected: Skid Resistance, pavement macro texture type of pavement surface, pavement age taking into account the last improvement, traffic level of the last six years and weather geographic area classified as wet or dry. The first two variables were measured in-field and the last three variables were obtained from the national road inventory (NRI) and from the national traffic inventory (NTI).

All these variables were arranged in a factorial matrix to define the panels in which data were classified to assess their interactions and trends (See Tables 1a and 1b). A first specification of the boundaries of the skid resistance, macro texture, pavement age and traffic levels was assumed. During the statistical analysis, these boundaries were adjusted applying the k-means and dendrogram classification algorithms.

Pavement Surface	Weather Zone	Cumulated Equivalent Traffic Levels								
		Low			Medium			High		
		Pavement Age								
		1 - 4	5 - 10	11-20	1 - 4	5 - 10	11-20	1 - 4	5 - 10	11-20
Asphalt Concrete	Wet									
	Dry									
Double Treatment	Wet									
	Dry									
Concrete	Wet									
	Dry									

Table 1a: Factorial matrix for statistic assessment of long term trend

Pavement Surface	Weather Zone	Cumulated Equivalent Traffic Levels											
		Low				Medium				High			
		Season											
		Sp	Sm	F	W	Sp	Sm	F	W	Sp	Sm	F	W
Asphalt Concrete	Wet												
	Dry												
Double Treatment	Wet												
	Dry												
Concrete	Wet												
	Dry												

Sp: Spring; Sm: Summer; F. Fall; W: Winter

Table 1b: Factorial matrix for statistic assessment of seasonal trend

The sample size was obtained using the software GPower developed by Erdfelder et al (1996). The software is a comprehensive implementation of the methods of sample size estimation based on the power statistical analysis method developed by Cohen (1988). To obtain the sample size two criteria were used: a medium size effect and high statistic power. The former permits to control of the effect of the sample size over the inflation of variance and the gap between the correlation coefficients of the sample and of the population of data. Higher statistic power permits to take control of the type II and type I errors in statistical analysis. With these considerations, a minimum sample size of 55 sample units in each cell of the factorial matrix was obtained.

The sampling method was based in the window method applied early by Videla et al (1996) to calibrate IRI deterioration models at network level in Chile. The method assumes that the test sections that have the same age, pavement surface and traffic level are homogeneous and independent one of each other. It means that no cross-effect exist between each test section. To assemble all the set of homogeneous test sections in a single process, at least the weak ergodic hypothesis need to be accomplished, as explain Echaveguren (2008).

The selection of test sections was based on the NRI of the MOP and in the factorial matrix depicted on Table 1. A preliminary selection of test section from the NRI was done considering pavement condition, location in relation to the headquarters of NHL, weather conditions, traffic conditions and pavement age. Then, the research team assess each test section in field to verify the pavement surfaces condition, traffic level, lateral areas of the roads, and safety conditions. A cross check between the detailed NRI and in-field assessment of each test section was done. From this comparison, a definitive set of test section was selected to perform the measurements. A summary of the number of test sections can be seen in Figure 1.

Pavement Surface	Weather Zone	Cumulated Equivalent Traffic Levels								
		Low			Medium			High		
		Pavement Age								
		1 - 4	5 - 10	11-20	1 - 4	5 - 10	11-20	1 - 4	5 - 10	11-20
Asphalt Concrete	Wet	37	54	78	64	64	91	30	80	81
	Dry	76	64	30	0	159	70	81	64	129
Double Treatment	Wet	71	144	77	32	42	77	0	0	0
	Dry	31	112	68	0	114	146	0	33	13
Concrete	Wet	0	0	26	0	0	45	0	0	103
	Dry	0	0	0	0	0	77	0	18	108

Pavement Surface	Weather Zone	Cumulated Equivalent Traffic Levels											
		Low				Medium				High			
		Season											
		Sp	Sm	F	W	Sp	Sm	F	W	Sp	Sm	F	W
Asphalt Concrete	Wet	34	34	34	34	47	47	47	47	33	33	33	33
	Dry	10	10	10	10	66	66	66	66	80	80	80	80
Double Treatment	Wet	20	20	20	20	15	15	15	15	0	0	0	0
	Dry	59	59	59	59	61	61	61	61	0	0	0	0
Concrete	Wet	17	17	17	17	20	20	20	20	0	0	0	0
	Dry	0	0	0	0	24	24	24	24	15	15	15	15

Sp: Spring; Sm: Summer; F. Fall; W: Winter

Figure 1. Number of Test Section selected for the Experiment

2.2 DATA COLLECTION AND PROCESSING

Data were collected between 2005 and 2007. Seasonal data were collected in a range of days representative of each season considered in the factorial matrix of Table 2.

SR data were collected using the SCRIM device and Macro Texture data were collected using Laser Profiler. Both kinds of data were collected each 5 meters. For data processing, the measurement procedures described in De Solminihac et al (2007) were used.

The main tasks of the procedure for measurement and data processing were the follows:

(a) calibration and assess of the repeatability to ensure that the devices are working well,
(b) detection of invalid data, to ensure that the raw data were relevant for modelling,
(c) adjustment of SR data to a standard value measured at 50 Km/h and at a weather temperature of 20°C, using adjustment factors developed by Echaveguren (2008),
(d) for a preliminary characterization of SR and MTX data, structural changes and outliers data were identified using the procedure of De Solminihac et al (2008). Mean and standard deviation of SR and MTX variable was obtained for un-equal length segments,
(e) to built the sample units, the data segmented on step (d) were superimposed to a systematic segmentation based on 100 m length segments,
(f) Each sample unit of 100 m length obtained contained mean and standard deviation values of SR and MTX,

3 EXPLORATORY DATA ANALYSIS

Exploratory Data Analysis (EDA) had the objective of to know the general trends of the variables involved in the study, mainly skid resistance and macro texture, on the Chilean road network. EDA considers descriptive statistic analysis, goodness of fit, clustering of data, box plot analysis, and analysis of variance. EDA was performed with aid of statistical software package MINITAB14.2. The results of each analysis are showed in the next sub sections

3.1 EDA FOR MACRO TEXTURE DATA

Macro Texture data was first grouped in clusters using K-mean method. Clustering considers three levels, low, medium and high. Once the individual data were assigned to each level, a goodness of fit test was used to define the probability distribution function (PDF) of data in each subset. The goodness of fit was performed using the Jarque – Bera (1987) test according the procedure implemented by Lawford (2005). In all the cases, the PDF obtained was a Gaussian function.

Figure 2 shows that the mean value of MTX ranges between 0,4 to 1,5 mm (expressed as SMTD units), but the minimum value obtained in the network was 0.2 mm and the maximum 2.6 mm. In Chilean network, normally the lowest values of

MTX are associated to very dense asphalt concrete mixes. Moreover, in double treatments with aggregates very polished and uncrushed or with aggregated covered by thin asphalt binders, is possible to find MTX values ranging between 0.3 mm and 0.7 mm.

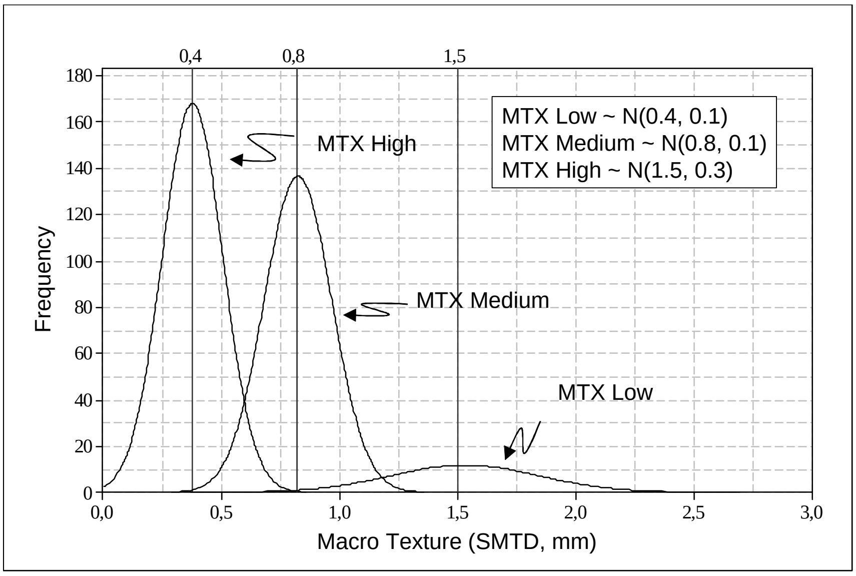

Figure 2: Summary of Descriptive Statistics of Macro Texture Data.

The box plot of MTX data of the Figure 3 show a slight trend to rise MTX values in double treatment, but is not possible to talk about that as general rule. In fact, the differences between the mean values of MTX against the type of pavement surfaces are lower than 0.3 mm in each level of MTX.

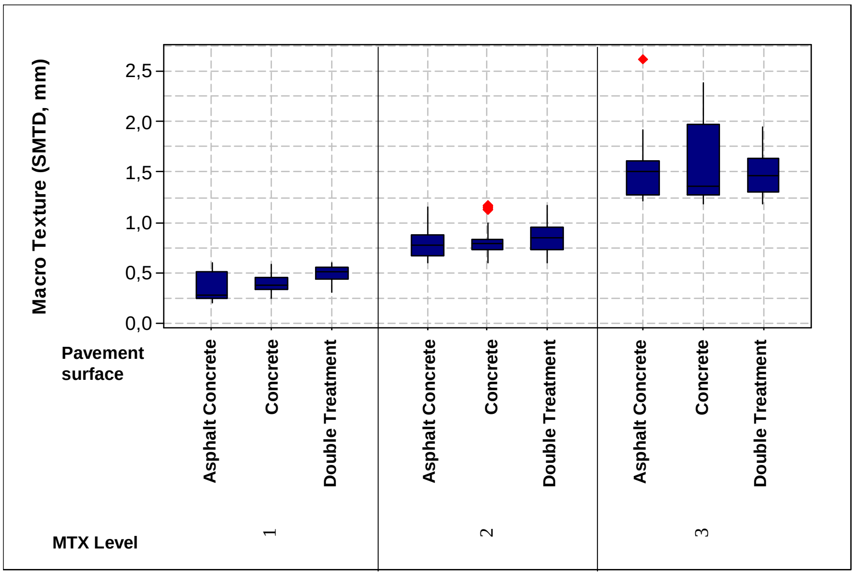

Figure 3. Box plot of MTX against type of pavement surface and macro texture level.

An assessment of the effect of factors: pavement age, weather zone, and traffic, over the MTX were done. The objective was to define whether the MTX behaves like a dependant or independents variable for modelling purposes. The analysis is summarized in Figure 4. The figure shows that the traffic, weather zone, and pavement age have not effect over MTX. In all these cases, the effect is lower than 0.1 mm. Since MTX is not depending of those factors, it can be considered as independent variable.

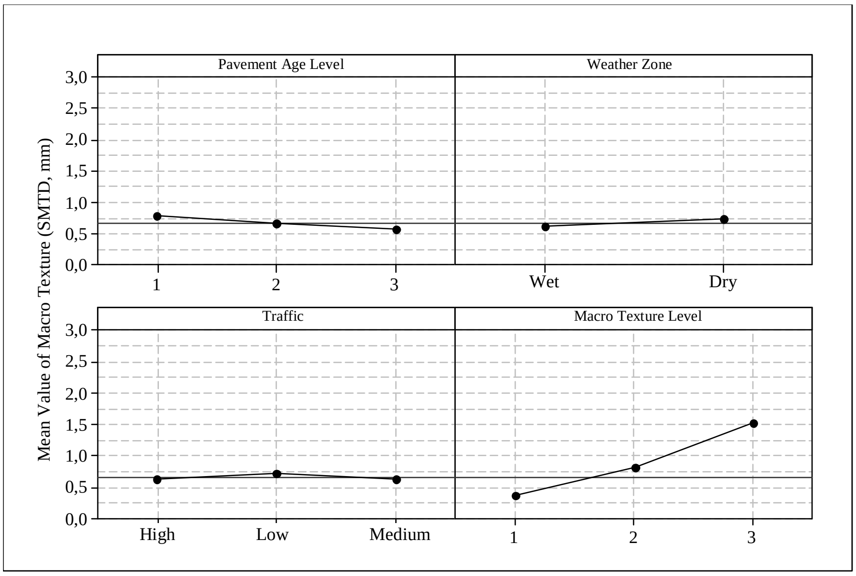

Figure 4. Graphical Summary of Factor Analysis of MTX

3.2 EDA FOR SKID RESISTANCE DATA

Similarly, of MTX, EDA was performed for SR data. In this case, the whole data were considered to obtain descriptive statistics. A summary of the results are showed in the box plot of Figure 5.

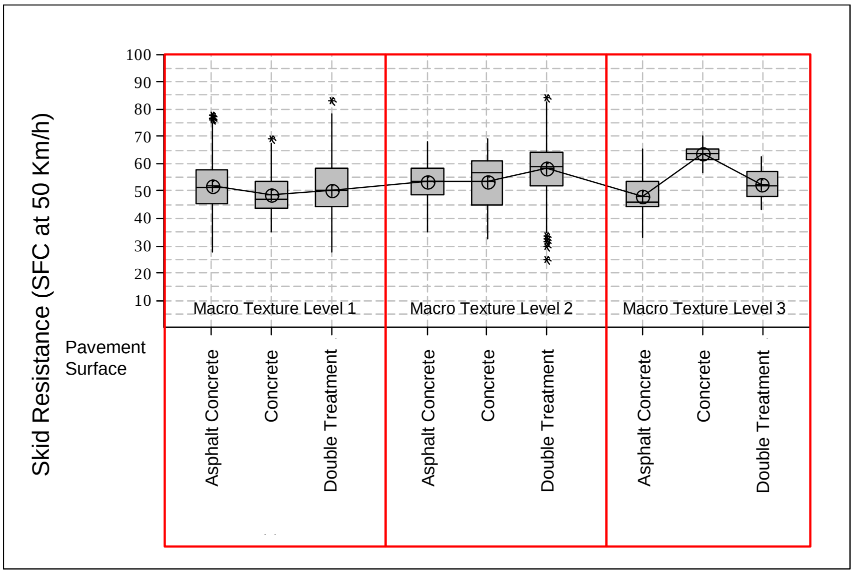

Figure 5: Box plots of SR values for each type of pavement surface surveyed.

The box plot shows that the SR values ranges between 0.95 and 0.25 (measured with SCRIM at 50 Km/h), but this values are isolated case. The mean values for all the surfaces is approximately 0.53 and the standard deviation is 0.09. Also, it can be seen that the SR values are not dependant of the macro texture level. Because the values are adjusted to a standard temperature, the differences between SR individual values can be explained by the different micro texture of the aggregates presented in each test section.

The factor analysis of the SR is summarized in Figure 6. In that figure can be seen that the pavement age and the traffic level affect the mean value of SR. The pavement age panel show that in older pavements (age level 1) the SR is lower than de SR mean value measured in new pavements (age level 3).

The traffic panel shows a similar behaviour. In high trafficked pavement, the SR is low and in low trafficked pavements, the SR is high. It follows that the findings are consistent with the behaviour expected and reported in the international literature.

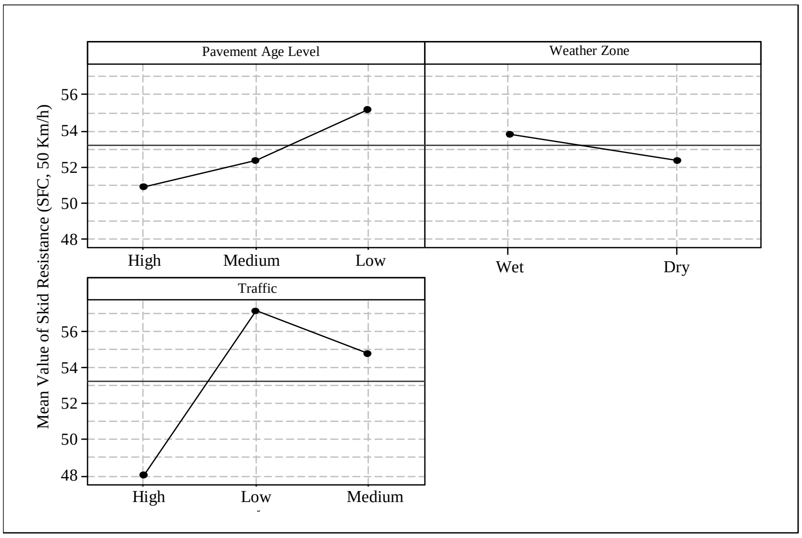

Figure 6. Graphical Summary of Factor Analysis of SR

3.3 PROOF OF WEAK ERGODICITY

Ergodicity is a desirable property of stochastic process. It means that the ensemble average of the realizations of the process is equivalent to the time average (Taylor and Karlin, 1998). In this case, is possible to extract “windows” of data from one realization of the process at any time. If the process is ergodic, that window is representative of the stochastic process.

In addition, all these windows can be assembled to obtain a representation of the stochastic process from snapshots of several different realizations. It follows that at least weak ergodicity is a necessary condition to apply the window sampling method used in this research. To proof the ergodicity in a practical sense, many authors recommend studying the partial autocorrelation function (PACF) of the process (Taylor and Karlin, 1998). If the PACF tends quickly to 0 when lag parameter tends to infinitum, it says that the process is weakly ergodic.

To proof that condition, a moving average model of first order (MA(1)) was built with SR and pavement age data and from this model the PACF was built. The behaviour obtained of the PACF for this model is showed in Figure 7.

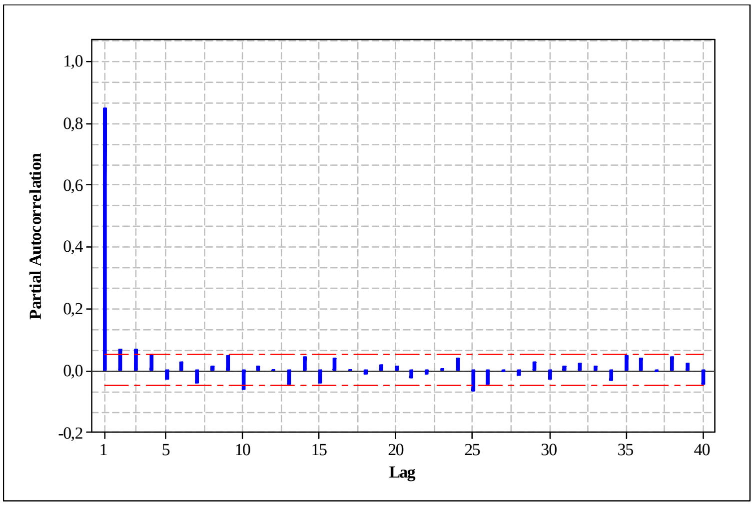

Figure 7. Plot of Partial Autocorrelation Function against lag parameter.

The figure shows that the PACF tend to 0 when the lag is higher than 5. It follows that can be assumed that the SR process for the whole network surveyed is weakly ergodic, and therefore the sample method used is valid.

4. TREND AND SEASONAL BEHAVIOR MODELS

4.1 CONCEPTUAL MODEL

The conceptual model is composed by 4 main modules, related to measurements, traffic and SR behaviour models, as shown in Figure 8. There is not the objective of the paper to explain in detail the mathematical modelling, but the general trends obtained in the analysis performed. For this reason, only is presented a general description of each model.

- Module 1: Skid Resistance Measurements Model. This model had the objective to obtain accurate SR data for modelling adjusted by temperature and speed factor developed. It was explained in section 2.2 of this paper.

- Module 2: Traffic Model: This model estimates the cumulated equivalent traffic (CET). The model considers the probability of that one vehicle wheel passes over the same track line by mean of wandering estimation. Also, estimate the damage factor applying the Brush's model to estimate the tyre-road horizontal forces for 4 types of vehicles. The result of the model is a history of CET in all the test sections.

- Module 3. Macro Texture Measurements Model. This model had the objective of to obtain accurate measurements of macro texture. To apply this module, MTX data were measured using laser profiler method and afterward the data processing method of section 2,2 was used to obtain data for modelling.

- Module 4: Skid Resistance Models: SR models are classified in trend and seasonal models. Trend models were calibrated in a span of pavement age of 1 to 20 years. Seasonal models were calibrated using 1-year data monitored each season.

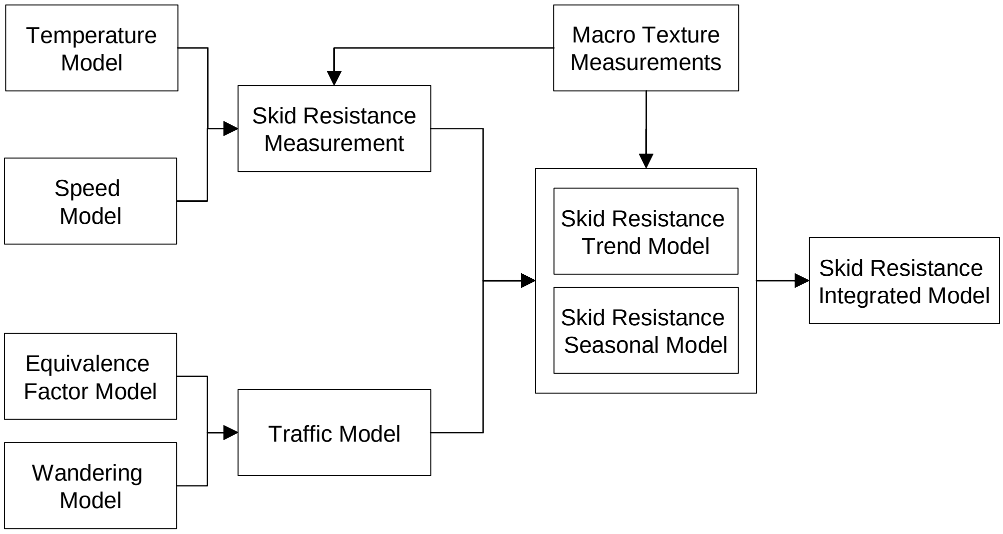

Figure 8. Conceptual model of skid resistance behaviour over the time.

4.2 ANALYSIS OF SEASONAL AND TREND MODELINGS

4.2.1 Traffic Model Trends

The CET data were clustered in 3 levels: high, medium and low for each pavement age ranging between 1 to 20 years. Only the last 5 years showed statistical differences between clusters. Therefore, the clusters were defined based on the last 5 years of traffic data. Figure 9 shows the models obtained for each CET level. The correlation coefficient of the 3 models ranges between 0.94 and 0.99. The models calibrated are valid for the whole road network surveyed in absence of congestion and relevant traffic reassignment.

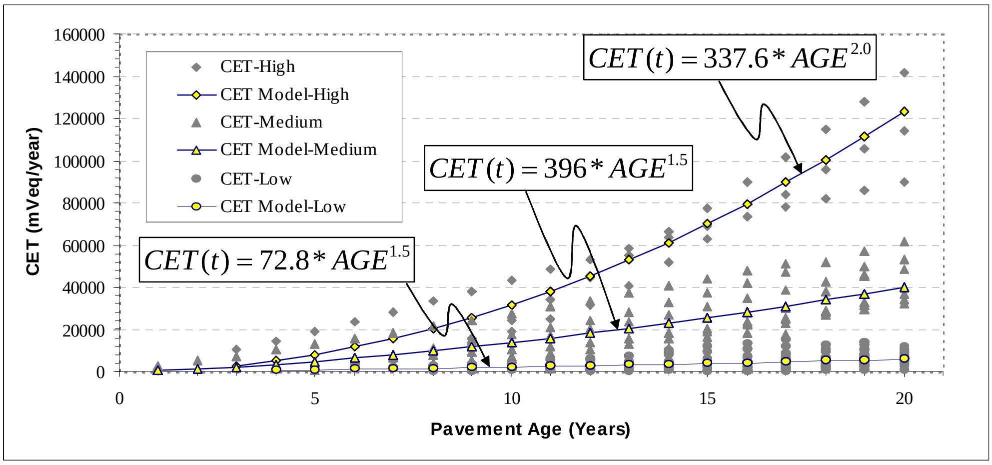

Figure 9. Cumulated traffic models for three traffic levels.

The trends observed in the network and the estimations of the models are logical and consistent. The low volume roads exhibit very low variations over the time and in consequence, it can be expected that the skid resistance does not vary too much over the time.

4.2.2 Skid Resistance Time Dependant Model

Five models were calibrated. Two models for high and low macro texture, 2 models for concrete and asphalt surfaces and one model for all the pavement surfaces as a whole.

Trials with the Hoerl, Weibull and Potential mathematic specifications were done to identify a consistent model from a physical and statistic point of view. Robust non-linear regression was used to analyze these models. The Weibull model of Eq. 1 was chosen for a detailed calibration. In that expression A, B, C and D are calibration parameters. CET variable was defined in section 4.2.1

$$SR = A + Be^{\left[\frac{C}{CET^{D}}\right]} \quad (1)$$

The standard error obtained with this model ranges between 0.01 and 0.06 and the correlation coefficient ranges between 0.6 and 0.95. Figure 10 shows the different types of models calibrated and validated.

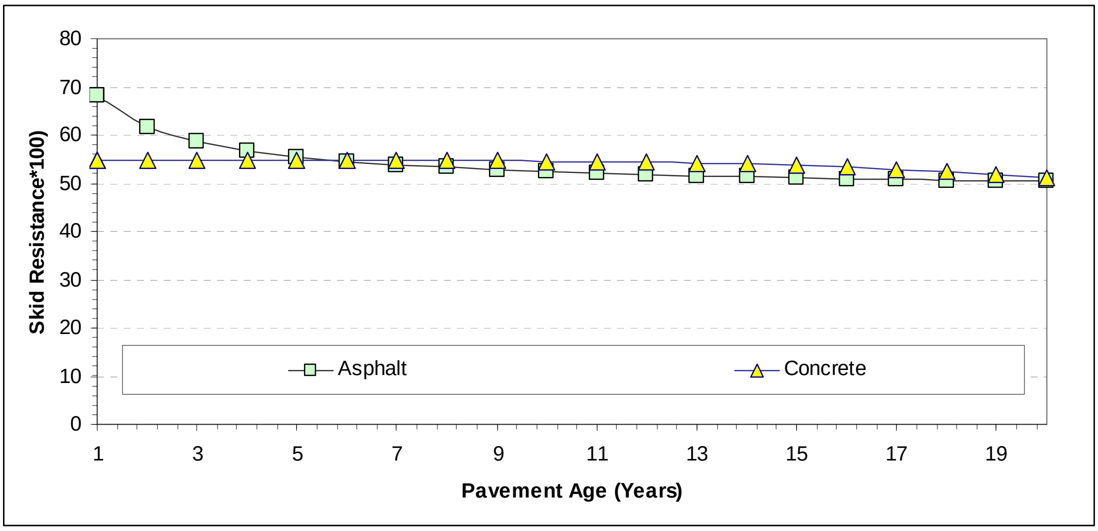

Figure 10: Skid resistance trend models for asphalt and concrete surfaces.

The models shows trend patterns depending on the growth rate of the traffic, traffic level, type of pavement surface, that provide interesting tools to manage skid resistance at a network level. The findings can be summarized as follows:

– Concrete pavement behaviour was more stable than the asphalt surfaces. The reduction of skid resistance in concrete surfaces in the first 5 years was not significant. In contrast, in asphalt surfaces skid resistance fall in 13 units in the first 5 years.

– The skid resistance drop at a mean rate from 0.01/year in low volume roads to 0.05/year in high volume road both in the first 5 years.

– In low volume road whether a maintenance program each 5 or 10 years is complimented with higher SR values in new pavements, the lowest SR value that can be obtained can be approximately a 40 % higher than a minimum threshold of 0.35.

4.2.3 Skid Resistance Seasonal Model

To calibrate these models a sinusoidal mathematic specification was used. This specification is showed in Eq. 2. In this equation, SI is a seasonal index that compresses 3 months in a single number. SRB is the skid resistance around the oscillation occurs, A0 is the amplitude, w is the wavelength and D is the starting lag of the wave.

$$SR(SI) = SRB + A0\cos(wSI + D) \qquad (2)$$

In contrast with the models founded in the literature, the span of time in which 1 period is raised was not 1 year of the Julian calendar. On this way, the period of the oscillatory wave was obtained directly from the calibration process.

The model was calibrated and validated considering: three types of pavement surfaces, and two levels of macro texture. In all cases, robust non-linear regression method was used. The standard errors range between 0.04 and 0.11. The lower value was obtained when modelling concrete surfaces. The higher error was obtained in double treatment – low macro texture models. Figure 11shows the different types of models calibrated and validated.

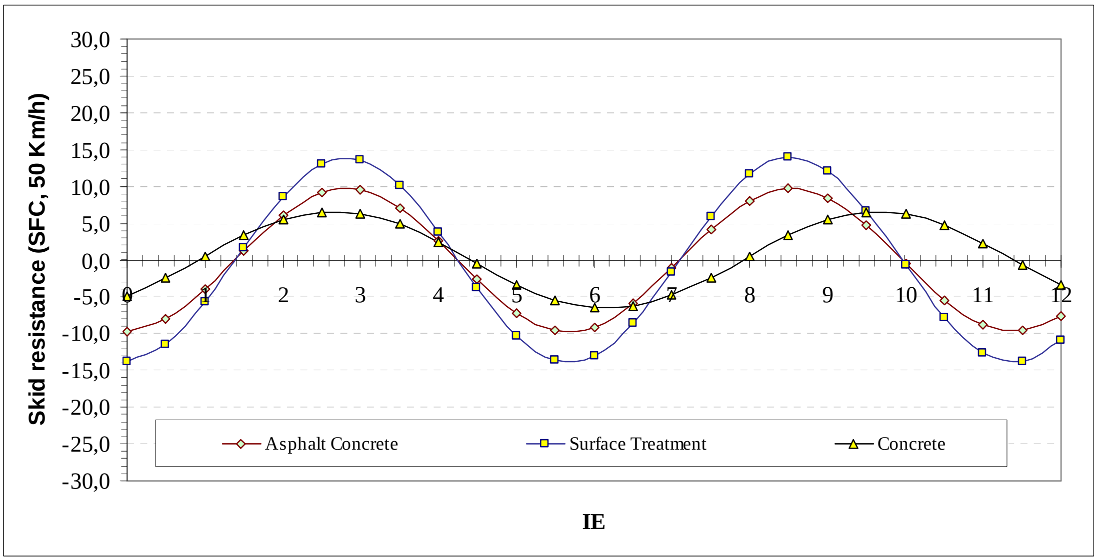

Figure 11: Skid resistance seasonal models as function as type of pavement surface

Seasonal oscillation is mainly affected by the interaction between the type of pavement surface, weather and pavement temperature. Concrete surfaces exhibit a more stable behaviour over the time and season-by-season, independently of the traffic level, weather and pavement temperature. In this kind of surfaces, amplitude obtained was 0.07. In asphalts surfaces the amplitude range from 0.1 to 0.15.

4.2.4 Skid Resistance Integrated Model

Both the trend model and seasonal model were integrated to assess the behaviour of Chilean paved road network surveyed. In Figure 13, an example for three types of pavement surfaces is showed.

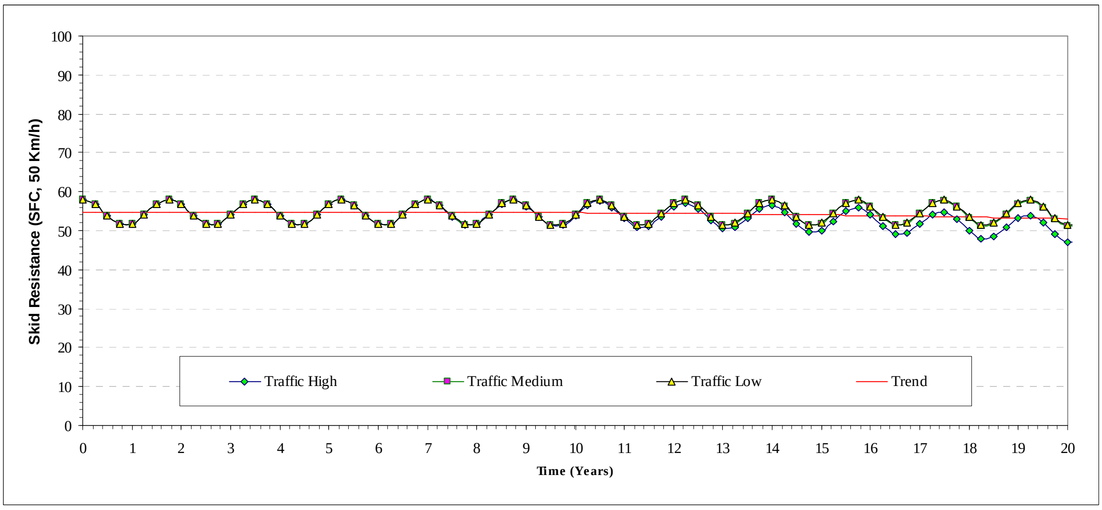

Figure 12a: Skid resistance integrated model. Concrete Surface.

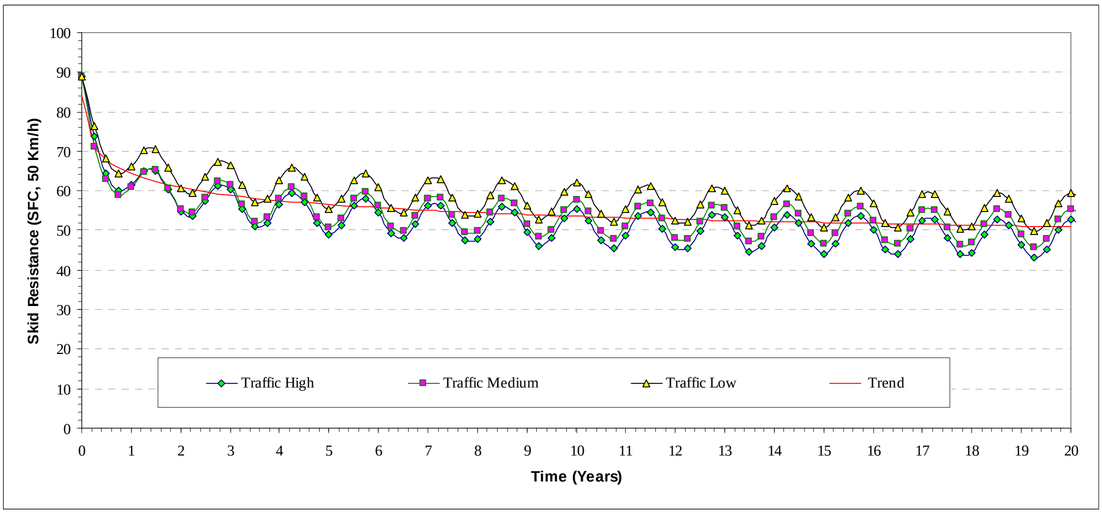

Figure 12b: Skid resistance integrated model for Double Treatment Surface.

Paved low volume roads in Chile have double treatments surfaces and a few roads asphalt concrete surfaces. In both cases in the first years (or immediately after the last improvement of the surfaces), the maximum value observed was 0.9. The integrated model shows that the lower value of skid resistance was near to 0.4 in a span of pavement age of 20 years.

Medium and highly trafficked roads are mainly paved with dense asphalt concrete mixtures. In this surfaces, the SR values range from 0.72 in the first years to 0.4 in 20 years. These results are more representatively of medium and high trafficked paved roads.

The age of concrete pavements ranges between 11 to 20 years and medium traffic levels. In that surfaces the values of SR can range between 0.6 and 0.5. This range of values can be considered as equilibrium values in a wide sense, considering that these values represent the hill and valleys of the oscillatory wave.

In concrete surfaces, the main variation of skid resistance is explained by the seasonal oscillation. Otherwise, in asphalt surfaces approximately the 50 % of the loss of skid resistance in the first years is explained by the seasonal oscillation only when the pavement is put in service on winter. In this case, a delay between 3 or 9 months in the start of loss of skid resistance was estimated with the models.

5. CONCLUSIONS

A basic task to understand the behaviour of the SR and MTX in a road network is to carry on a survey plan that permits to obtain representative data from a statistical and spatial point of view. For design surveys plan, the statistical experimental design and the “windows” sampling method are useful tools that complimented with in-field inspection permits to identify suitable test sections for monitoring. Therefore, the sampling method based on windows only can be used if the stochastic processes that describe skid resistance over the time have the property of weak ergodicity. For this reason is recommendable before to apply the full plan of surveying, to evaluate in a small subset of the network that property.

Skid resistance data have high variability that depends on the weather conditions, pavement conditions and the measurement device among others. For modelling purposes, the measurement, processing and analysis of data is more rigorous than the process to obtain data for managing. A strict quality control for measurement is necessary to reduce variability induce for instance for the measurement speed variation and trajectory oscillation. In addition, a data processing method to identify outliers, invalid data and structural changes permits to build sample units with skid resistance and/or macro texture data suitable for modelling.

Cumulated traffic is the most relevant variable that explains the behaviour of skid resistance over the time. For this reason, in the research a specific model was developed that improves the method for calculate the damage factor proposed in the literature using the Brush model. Moreover, in order to achieve a more detailed an accurate explanation of the damage effect of the traffic is necessary to develop explicit relationships between this damage factor and the quality of the aggregates.

Skid resistance models are useful tools to understand general trends and behaviours at a network level. Considering the high amount of data necessary to collect to an adequate study, a model is a better tool of synthesize. In this way, models not only permits to understand behaviours, but permits to have a useful basis for planning monitoring and to estimate thresholds at network level.

The research described in this paper was the first full-scale study conducted in Chile to characterize skid resistance and macro texture data in the paved road network This study was the knowledge base to update the national skid resistance standards and for improve the state of the practice related to measurement and quality control.

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REFERENCES

Cohen, J. (1988). Statistical power analysis for the behavioral sciences. 2nd Edition. Academic Press.

De Solminihac, H. Echaveguren, T. and Bustos, M. (2006). Assessment of time dependant models of pavement skid resistance. Construction Engineering Journal (RIC), 21(3), 157-168.

De Solminihac, H. Echaveguren, T. Chamorro, A. Vargas, S. y Bustos, M. (2007) *Procedimiento de Medición y Procesamiento Datos de Resistencia al Deslizamiento Medidos con Equipo SCRIM*. Technical Document TM-014-06-2. Research Project FONDEF D03I-1042. Pontificia Universidad Catolica de Chile.

De Solminihac, H. Vargas, S. and Echaveguren, T. (2008). Proposal of a segmentation procedure for skid resistance data. Arabian Journal of Science and Engineering. (To be published in April 2008).

Echaveguren, T. (2008). Modeling of pavement skid resistance. PhD Thesis. Pontificia Universidad Católica de Chile. Chile.

Erdfelder, E. Faul F. and Burchner, A. (1996). GPOWER: A general power analysis program. Behavioral Research Methods, Instruments & Computers, 28, 1 – 11.

Jarque, C and Bera, A (1987). A test for normality of observations and regression residuals. International Statistic Review, 55(2), 163 – 172.

Lawford, S. (2005). Finite-simple quantiles of the Jarque-Bera test. Applied Economic Letters, 12, 351 – 354.

Taylor, H. M. and Karlin, S (1998). An introduction to Stochastic Modeling. 3rd Edition. Academic Press. London.

Videla, C De Solminihac, H Gaete, R and Bustos, M (1996). Ajuste de factores de calibración para ampliar modelos de deterioro en pavimentos asfálticos. Proyecto DGOP-190-08-03-1994. DICTUC. Pontificia Universidad Católica de Chile. Chile.